The effects of different Cooling Materials and Windows

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Abstract

The equilibrium emittances and rates of cooling are calculated for different materials, windows and foils. A comparison with ICOOL shows that these calculations give good estimates of simulated cooling in a given lattice. Assuming that the use of hydrogen will require a second safety window twice as thick as the containment window, then it is shown that the performance of hydrogen is almost the same as helium with no such safety window, and very little better than that with lithium hydride and no containment.

ICOOL simulations are shown for the full study 2 cooling channel, with different window and absorber materials. The addition of the safety window reduces the $\frac{\mu}{p}$ ratio by 9%. The use of helium (without safety window), or LiH (with no window) give the same performances: about 5% less than that for hydrogen with safety.

It is also shown that grids of tubes (80% coverage) with 25 μm (1 mill) aluminum walls would give the same scattering as the study 2 beryllium windows.

1 Introduction

To meet safety requirements, one, or even two, stronger safety windows may be required between the hydrogen (H) absorbers and the rf. This note explores the consequences of adding such material and looks at alternatives of helium (He) and lithium hydride (LiH).

Table 1: Energy loss, radiation length, Q values and other parameters for considered absorber materials and windows. [2]

Material	K_{cond} (J/s m K)	T_{melt} (C)	dE/dz (MeV/cm)	L_R (cm)	Q(mat) (mm mrad/cm)
Liq. H_2			0.286	865	38
Liq. He			0.242	755	51
Li H	8-14	677	1.59	97.1	61
Be		1261	2.95	35.3	89
Al	210	660	4.36	8.9	238

2 Calculation

Using the energy loss dE/dz, radiation length L_R, the betatron function $\beta_\perp$, and thicknesses of materials in a cooling channel, we can calculate the expected equilibrium emittances. If only one material is present then the equilibrium emittance is [1]:

$$\epsilon_{\rm equilib} \;=\; \beta_\perp \; F(E) \; Q({\rm mat}) \tag{1}$$

where

$$F(E) \;=\; \left(\frac{1}{\beta_v \frac{(dE/dz)}{(dE/dz)_{\rm min}}} \right) \quad , \quad Q({\rm mat}) \;=\; \left(\frac{{\rm E_s^2}}{2\ {\rm m_\mu L_R (dE/dz)_{min}}} \right), \tag{2}$$

and $E_s \approx 14.1$ MeV, $m_\mu = 105$ MeV, and all other units are mks.

$F(E)$ for our cases is close to unity. Values of Q(mat), and other constants, are given in Tb. 1.

For a channel containing several different materials with different lengths and local $\beta_{\perp i}$'s, one can define an effective Q'(mat) as a function of the individual lengths, $\beta_{\perp i}$'s, L_{Ri}'s and $(dE/dz)_i$'s:

$$Q'({\rm mat}) \;=\; \left(\frac{{\rm E_s^2} \beta_\perp}{2\ {\rm m_\mu}} \right) \; \left(\frac{\Sigma_{\rm i}^{\rm n} \left(\frac{\beta_{\rm i}}{\beta_\perp} \right) \frac{1}{\rm L_{Ri}} \Delta \ell_{\rm i}}{\Sigma_{\rm i}^{\rm n} \left(\frac{\rm dE}{\rm dz} \right)_{\rm i} \Delta \ell_{\rm i}} \right) \tag{3}$$

For an extended channel with average momentum loss $\frac{dp}{d\ell}$

$$\epsilon_n \;=\; \epsilon_{\rm equilib} \;+\; (\epsilon_{\rm initial} \;-\; \epsilon_{\rm equilib}) \; e^{\frac{dp}{d\ell}\frac{z}{p}} \tag{4}$$

Table 2: SFOFO lattice parameters used in the simulations of the MICE experiment.

lattice	(1,1)	(2,3)	
cell	2.75	1.65	m
F(E)	1.05	1.05	
p_{ave}	220	213	MeV/c
ϵ_o	8.9	2.2	π mm rad
Δp	13.14	7.9	MeV/c
$\beta_\perp$	40	18	cm
$\beta_{rf}/\beta_\perp$	2	3	

and the initial rate of emittance reduction is seen to be:

$$\left(\frac{1}{\epsilon_n}\frac{d\epsilon_n}{d\ell}\right)_{\text{initial}} = \frac{1}{p}\frac{dp}{d\ell}\left(1-\frac{\epsilon_{\text{equilib}}}{\epsilon_{\text{initial}}}\right) \tag{5}$$

3 Comparison with ICOOL

The following plots show the cooling calculated from the above formula, and as simulated by ICOOL [3] for the SFOFO lattice being studied for the cooling experiment (MICE) [4]. One cell of the example uses 35 cm of liquid hydrogen, two 360 μm aluminum windows, and a total axial thickness of 2.5 mm (0.2 + 3×0.7 + 0.2) of beryllium for the rf window. The parameters used were as given in Tb. 2. For comparison, those for the final Study 2 [5] cooling lattice are also given.

From the plot, it is seen that the theoretical prediction of cooling is well reproduced by ICOOL.

4 Grids *vs.* Foils

At the start of the cooling channel there is a total of 2.5 mm of Be foils per cell. This is the thickness on axis. In study 2, the foils are stepped, so that at the outer edges there is twice this thickness, but relatively few particles pass through this outer region, and its effect is not significant.

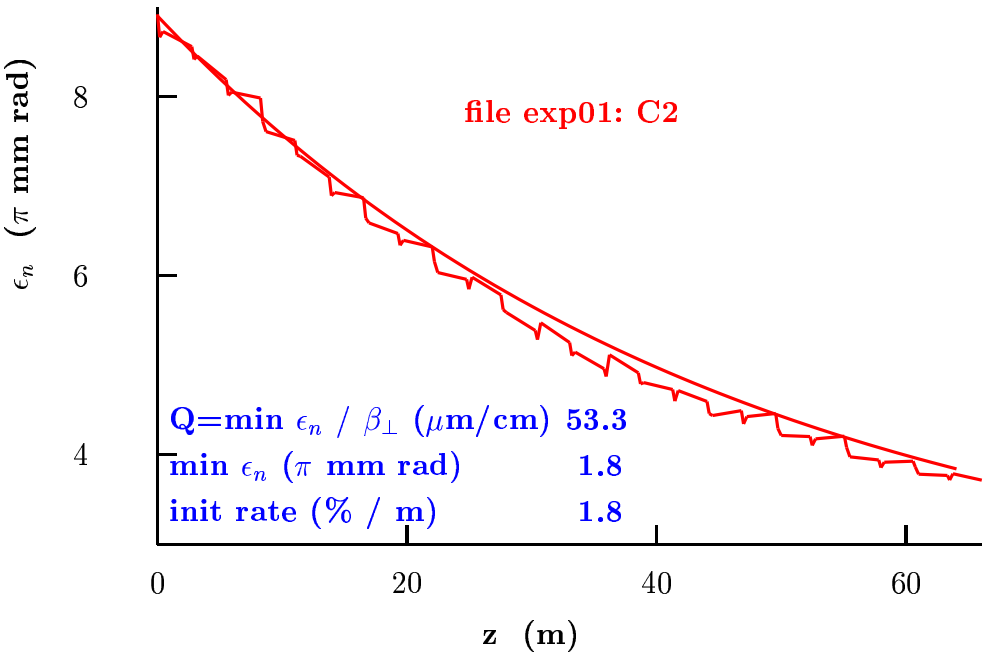
file exp01: C2
Q=min ϵ_n / $\beta_\perp$ (μm/cm) 53.3
min ϵ_n (π mm rad) 1.8
init rate (% / m) 1.8
ϵ_n (π mm rad)
8
6
4
0
20
40
60
z (m)

Figure 1:

If the center to center spacing of tubes [6] in a grid is $f_{\rm spacing} \times 2r$ then, on average, the wall thickness of aluminum tubes that would have the same scattering effect would be:

$$t_{\rm Al\ tube} = \frac{L_{\rm Al}}{L_{\rm Be}} \frac{f_{\rm spacing}}{\pi} \frac{\Sigma\, t_{\rm Be}}{n_{\rm grids}} \tag{6}$$

for $\Sigma\, t_{\rm Be} = 2.5$ mm (80% coverage), $f_{\rm spacing} = 1.25$, $n_{\rm grids}$=10 (two planes for each boundary of 4 cavities, then the equivalent wall thickness is 24 μm.

This is a somewhat conservative estimate, since it ignores the effects of the greater Be thicknesses at the outer radii. It is also conservative in that grids might only be used between cavities, with Be foils used at the ends where the apertures are smaller, the heating down, and the required thickness less.

The average scattering from such tubes is less if their spacing is wider, but the surface fields will then be higher (approximately linear with the spacings divided by the diameter. Even with the tubes touching, there will be a significant field enhancement.

5 Calculations for Different Cases

Given this agreement, we can make calculation for other cases. The following plot shows values of calculated minimum emittance for $\beta_\perp$=1 cm (Q'), as a function of the total thickness of aluminum windows, with LH, LHe and LiH. The continuous thin lines show results with Be rf windows. In the hydrogen case, the dashed lines show results without the rf windows, and the continuous thick line is for the case of 125 μm (5 mill) wall aluminum tubes, with 80% coverage, instead of the rf Be windows.

Particular values have been selected and marked on the plots and given in Tb. 3.

1. For hydrogen with no windows.

2. For hydrogen with the Be rf windows added.

3. For hydrogen with aluminum hydrogen windows, and Be rf, windows, but no *safety* windows. This is the case in Study 2.

4. As in study 2, but with aluminum safety windows twice the thickness of the primary windows, placed close to those primary windows. The

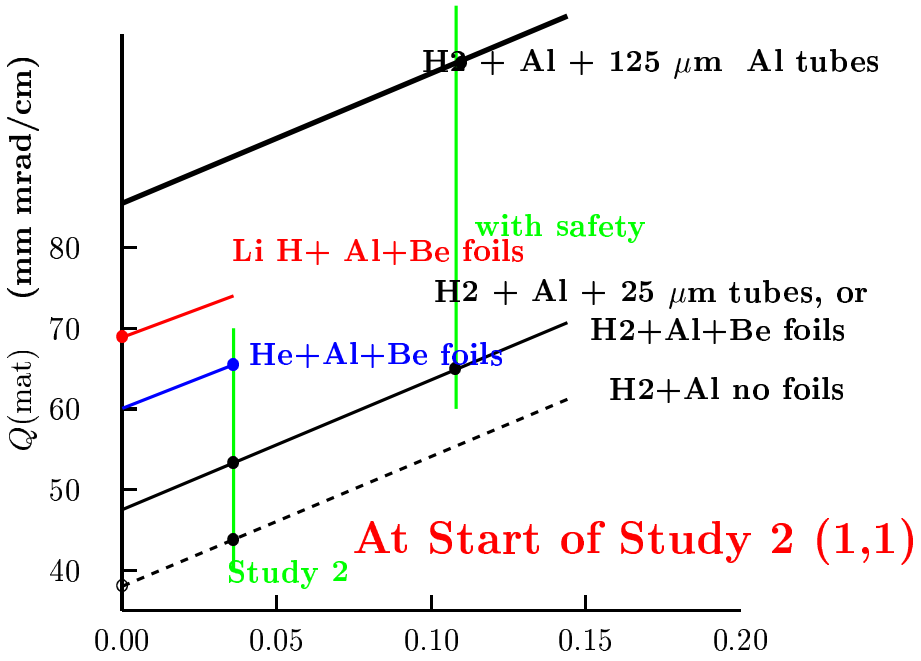

Figure 2:

doubling of the thickness corresponds to standard safety requirements, but their placement close to the primary windows does not. Larger, and thus thicker windows would be required if they were placed further from the hydrogen, and their effect would be greater because of rising $\beta_\perp$. On the other hand, it may be acceptable to make the safety windows of Be, which would reduce their effect.

5. For helium instead of hydrogen. The primary window is kept the same, but there is no safety window included.

6. For solid lithium hydride. No window is assumed. Some very thin window may be required to keep moisture out, but its effect is assumed to be insignificant.

7. For hydrogen with safety window, but 125 μm (5 mill) aluminum tubes instead of the Be foils.

We note that in all cases, the changes in cooling rate are far less than the changes in equilibrium emittance. This is because the channel $\beta_\perp$ is tapered so that the emittance is never allowed to fall near to this equilibrium.

Table 3: Parameters of the different cases discussed in the text.

Material	L1 cm	L2 cm	L3 cm	Q(mats) mm mrad/cm	$\epsilon_{\text{equilib}}$ mm	rate %/m
At Start	$\beta_\perp$=0.35 m			$\epsilon_{initial}$ = 8.9 πmm rad		
1) H2 only	19.3	0.000	0.000	38.0	1.29	1.95
2) H2 + rfw	18.0	0.000	0.125	47.5	1.61	1.86
3) H2 + Al + rfw	17.5	0.036	0.125	53.3	1.81	1.81
4) H2+Al+Safety+rf	16.4	0.108	0.125	64.9	2.20	1.71
5) He + Al + rfw	20.7	0.036	0.125	65.5	2.22	1.71
6) Li H + rfw	3.9	0.000	0.125	76.2	2.58	1.61
7) H2+Al+Safety+5 mill tubes	16.0	0.108	0.164	114.6	4.91	1.02

For the particular examples in the table, we note:

1. With only hydrogen and no windows, the equilibrium emittance is low (1.29 π mm rad), which is 1/7 of the assumed initial emittance. This leads to a cooling rate (1.95% /m) which is 6/7 of the ideal rate that would be obtained with no Coulomb scattering.

2. The use of Be rf windows raises the equilibrium emittance by 25%, but lowers the cooling rate by only 5%.

3. Adding the aluminum hydrogen windows raises the equilibrium emittance by 12%, and lowers the rate by 3%.

4. The addition of a safety hydrogen windows raises the equilibrium emittance by 22% (2.2 *vs.* 1.8 π mm rad), and lowers the cooling rate by 6% (1.71 *vs.* 1.81 %/m).

5. The equilibrium emittances and cooling rates for helium absorbers, assuming no need for safety windows, are the same as those for hydrogen with safety windows (1.71 *vs.* 1.71).

6. The use of solid lithium hydride, in this calculation for the initial cooling cells, raises the equilibrium emittances by 16% and reduces the rates of cooling by 6%. But the fact of the shorter lengths of LiH can compensate further down the cooling channel, where the $\beta_\perp$'s are lower.

Table 4: $*$ Note that the second case is not the same as that discussed above. It is for hydrogen and the study 2 Al windows, but no rf windows.

Case	ave $\frac{\mu}{p}$	Exp Rate %/m
1) H2 only	0.157 ± 0.06	1.95
2*) H2 + Al	0.148 ± 0.06	1.90
3) H2 + Al + rfw	0.139 ± 0.04	1.81
4) H2+Al+Safety+rf	0.127 ± 0.02	1.71
5) He + Al + rfw	0.121 ± 0.02	1.71
6) Li H + rfw	0.121 ± 0.02	1.61

7. The use of 125 μm (5 mill) aluminum tubes and 80% coverage, instead of Be foils raises the equilibrium emittance by a factor of 1.9 and lowers the cooling rate by 37%, which would presumably be unacceptable. Increasing the spacing by a factor of 5 would correct the problem, but increase the peak surface fields by the same factor.

6 ICOOL Simulations of Cases

To observe the effect of these cases on a neutrino factory, we made a number of simulations of the full Study 2 cooling channel. Each run used 5000 initial protons. Errors were estimated by observing the *rms* scatter of observed simulations using differing random starts. The resulting average $\frac{\mu}{p}$'s and their errors are given in Tb. 4, together with the calculated cooling rate for the first lattice.

The results are plotted below, and we see the same trend toward lower performance as the windows are introduced or increased in thickness.

It is interesting to compare these results with the calculated cooling rates for the first cooling lattice. One might expect a simple linear relationship. The following plot gives these μp results versus the calculated rates in the first cooling lattice. It is seen that there is indeed an approximately linear relationship between them, but that the performance with LiH is a lot better than expected, and that with He a little worse. These results can be understood (as suggested by Kaplan [7]) by the distance dependence of the $\beta_\perp$ from the center of the absorbers. This dependency $\beta_\perp = \beta_o + z^2/\beta_o$ is

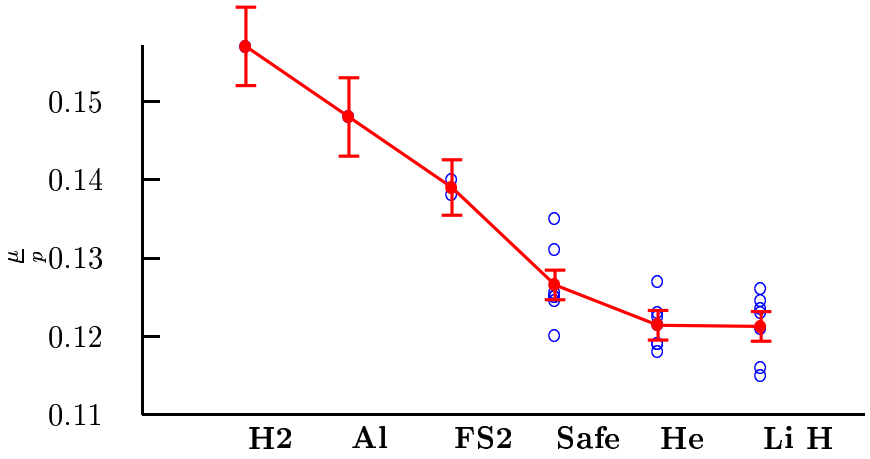

Figure 3:

weak for the first lattice where $\beta_o = 0.4$ m, but stronger for later lattices where β_o is progressively less. As a result, the dependence of performance on the thickness of the absorbers is greater for the full channel than for the first lattice. This effect favors the thinner LiH and disfavors the somewhat thicker He.

7 Conclusion

We need to think carefully about the costs, safety and inconvenience versus the need of using hydrogen in a neutrino factory, and consider the practicality of helium or lithium hydride as alternatives.

References

[1] Cooling theory chapter

[2] Particle Data

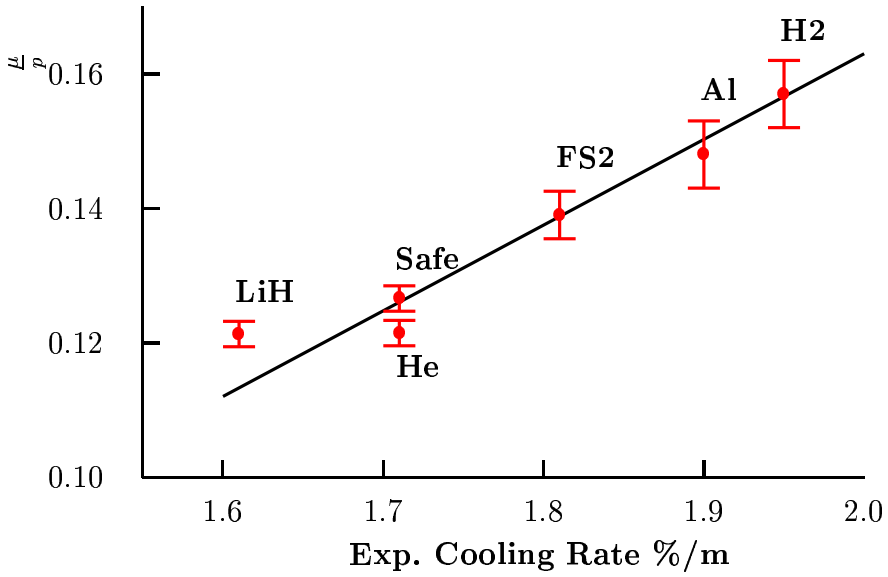

Figure 4:

[3] R. Fernow, ICOOL

[4] MICE experiment

[5] StudyII

[6] Chapter ...in reference [5]

[7] D. Kaplan, Technical Board Meeting, IIT, Chicago , Feb. 2002.